

**Numerical analysis of the dynamics
of a cosmic string loop as a vortex**

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ABSTRACT

Time evolution of a circular cosmic string loop is investigated by numerically solving the field equations for the scalar and the gauge fields consisting of the vortex. It is shown that the result agrees with an analytic estimate based on the Nambu-Goto action, which supports its validity in analyzing nonstraight and rapidly moving strings.

The cosmic string scenario is one of the major proposed formation mechanisms of density fluctuations in the universe [1]. Although it is true that quantum fluctuations produced during inflation provide realization of primordial density fluctuations more economically [2], cosmic string scenario is no less important in the present situation of cosmology that the type of the dark matter dominating mass density of the universe is yet unknown, because the latter can induce structure formation even in the hot-dark-matter-dominated universe in which linear density fluctuations on small scales are washed away due to free streaming. Note also that, contrary to the usual prejudice, it is easy to have cosmic strings compatible with inflation [3], which is anyway indispensable to realize globally homogeneous and nearly flat spacetime apart from the origin of fluctuations [4].

The most remarkable feature of the cosmic string scenario is that it has essentially only one parameter, namely, the line density of a string, μ . It should satisfy $G\mu \simeq 10^{-6}$ in order to meet various observational requirements of large-scale structures [5]. On the other hand, it also suffers from an upperbound imposed by the timing analysis of a millisecond pulsar [6], since the stochastic gravitational background radiation emitted by string loops can disturb it [7].

In the usual treatment of cosmic strings, their motion is described by the Nambu-Goto action, which is proportional to the surface area of the string world sheet [8]. Using the two dimensional metric tensor on the surface,

$$h_{ab} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \zeta^a} \frac{\partial X^\nu}{\partial \zeta^b}, \quad \mu, \nu = 0, 1, 2, 3, \quad a, b = 0, 1, \quad (1)$$

where $X^\mu(\zeta^a)$ is the spacetime trajectory of a string parametrized by a timelike parameter ζ^0 and a spacelike parameter ζ^1 , the action is expressed as

$$S = -\mu \int \sqrt{-\det h_{ab}} d\zeta^0 d\zeta^1. \quad (2)$$

The above expression can be obtained for the Nielsen-Olesen vortex line treating it infinitely thin and assuming the Lorentz invariance along the string [9]. Strictly speaking, therefore, the Nambu-Goto action is justified only for static and infinitely straight strings. The equations of motion deduced from the action would adequately describe dynamics of long strings which induce structure formation because their curvature scale is many orders larger than their thickness or the Compton wavelengths of the scalar and the gauge fields consisting of the vortex line. However, they may not correctly describe the motion of small loops, especially near cusp-forming regions, where velocity of the string reaches that of light.

Two approaches have been proposed so far to improve description of string motion. One is to calculate finite-width corrections to the Nambu-Goto action with respect to the ratio of the string width to the curvature radius [10]. The other is to employ a new phenomenological energy-momentum tensor for a string loop in which the line density and the string tension are allowed to take different values depending on position and time [11]. The latter research claims that these two quantities take considerably different values near a cusp forming region and that in some cases the line density vanishes at a cusp, which may imply snapping of a string loop. If this is the case, previous calculation of stochastic gravitational radiation background may have to be reconsidered.

In the present paper we investigate the validity of the Nambu-Goto action without resorting to perturbative or phenomenological methods. That is, we reproduce a string loop in terms of scalar and gauge fields and trace its motion by numerically

solving field equations. We compare a result of numerical simulation with an analytic calculation obtained solving the equation of motion given by the Nambu-Goto action. As a tractable example for the both analyses, we consider a circular loop.

In order to generate a vortex solution, we employ the Abelian Higgs model in the flat spacetime, whose Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial^\mu - ieA^\mu)\phi^*(\partial_\mu + ieA_\mu)\phi - V[\phi], \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ V[\phi] &= \frac{\lambda}{2}(|\phi|^2 - v^2)^2.\end{aligned}\tag{3}$$

The equations of motion for the complex scalar field $\phi(\mathbf{x}, t)$ and the gauge field $A_\mu(\mathbf{x}, t)$ read,

$$\left[\square + 2ieA^\mu\partial_\mu - e^2A_\mu A^\mu + ie(\partial_\mu A^\mu)\right]\phi + \lambda(|\phi|^2 - v^2)\phi = 0,\tag{4}$$

$$\partial_\mu\partial_\nu A^\nu - \square A_\mu = ie(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) + 2e^2A_\mu\phi^*\phi,\tag{5}$$

respectively. We take $\lambda = 0.01$, $e = 1$, and $v = 1$ and impose the Lorentz gauge condition, $\partial_\mu A^\mu = 0$, for numerical calculation.

In order to set up the initial configuration, using the relaxation method[12], we first reproduce the static and infinitely straight Nielsen-Olesen vortex [9], in which amplitudes of the scalar field and the gauge field are given as a function of the distance from the string core. Then it is bent artificially to make a circular loop. To be more specific, we calculate the minimum distance to the trajectory of the string core at each point and assign the corresponding field amplitudes just as the Nielsen-Olesen vortex, while varying the phase of the scalar field uniformly and the direction of the gauge field correspondingly. Since we also set $\dot{\phi} = 0$ and $\dot{A}_\mu = 0$ initially, where an overdot denotes time derivation, the second time derivative of both ϕ and A_μ is nonvanishing then due to bending. But it is small because we take the initial radius of a loop large compared with the core radius of a string.

Making use of the rotational symmetry, the above field configuration is embedded in the center of a square of 200^2 meshes perpendicular to the string loop with the boundary condition $\partial_i A^\mu = 0$ and $\partial_i \phi = 0$ where ∂_i denotes spatial derivatives. In fact only a quarter of the square is required for numerical computation due to the reflective symmetry. We take the core radius or $\frac{1}{\sqrt{\lambda v}}$ equal to $10\Delta x$ and the initial radius of the loop to be $50\Delta x$ with Δx being the mesh size. Then time evolution of the loop is traced solving equations. (4) and (5) in terms of the leap frog method with a constant time step $\Delta t = 0.1\Delta x$.

On the other hand, taking a cylindrical coordinate $ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + dz^2$ and $\zeta^0 = t$ and $\zeta^1 = \theta$, the Nambu-Goto action (2) for a circular loop is given by

$$S = -2\pi\mu \int R(t)\sqrt{1 - \dot{R}^2(t)}dt,\tag{6}$$

where $R(t)$ is the radius of the loop. Then the equation of motion reads

$$\ddot{R}(t)R(t) - \dot{R}^2(t) + 1 = 0,\tag{7}$$

which is easily solved as

$$R(t) = R_0 \left| \cos \left(\frac{t}{R_0} \right) \right|, \quad (8)$$

where we have assumed the loop is at rest with radius R_0 initially at $t = 0$. The speed of the loop motion is given by

$$|\dot{R}(t)| = \left| \sin \left(\frac{t}{R_0} \right) \right|, \quad (9)$$

which reaches the speed of light at $t = \frac{\pi}{2} R_0 \simeq 78.5 \Delta x$ ($R_0 = 50 \Delta x$), when the loop shrinks to a point.

Now we are in a position to display our result of numerical calculation and compare it with the above analytic estimate (8). Figure 1 depicts time evolution of the potential energy density, $V[\phi]$, of the scalar field up to the stage the loop shrinks completely. Due to the initial finite second derivative, the string loop is accelerated and its radius approaches zero. Figure 2 shows evolution of scalar field amplitude in terms of $F(\mathbf{x}, t) \equiv 1 - |\phi(\mathbf{x}, t)|^2$, which vanishes at the potential minima and is equal to unity at the string core. As is seen there the field configuration starts to be disturbed around $t \simeq 60$ and after the loop has shrunk to a point, it dissipates all the energy into scalar and vector waves without bouncing, which is seen in the further simulation. This is quite similar to the fate of a cylindrical domain wall consisting of a real scalar field [13].

Finally we compare time evolution of the trajectory of the string core $\phi(\mathbf{x}, t) = 0$ with the analytic result (8), which is summarized in Table 1. As is seen there the result based on the Nambu-Goto action agrees with the numerical calculation remarkably well up to the stage that the loop radius becomes as small as the core radius and that the speed of string motion is as large as 99% of that of light. We can interpret this result as a strong support to conventional analyses based on the Nambu-Goto action. We also note that the phenomenon that the string loop dissipates all the energy without bouncing has occurred simply because we have employed a too idealized configuration of a circular loop. It could be avoided if we would consider non-selfintersecting loops. Unfortunately, however, it is a formidable task to reproduce such a loop in terms of the scalar and the gauge fields themselves.

In summary, we have calculated time evolution of a circular cosmic string loop by directly solving field equations for the scalar and the gauge fields. The result agrees well with the analytic estimate based on the Nambu-Goto action, so that we may conclude that the action is applicable even for microscopically nonstraight strings moving with a relativistic speed.

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Figure Captions

Fig. 1 Time evolution of $V[\phi]$ on the 100^2 2-dimensional slice of the string loop whose radius is equal to $50\Delta x$ initially. The upper right corner corresponds to the center of the loop and r depicts the radial direction of cylindrical coordinates. These figures shows the epochs $t = 0, 30, 60, 70$ respectively.

Fig. 2 Time evolution of $F(\mathbf{x}, t)$ on the same slice for the same string as Fig. 1 at $t = 0, 30, 60, 70, 120, 160$.

Table 1

| Time | Loop Radius | | Speed |
|------|-------------|----------|-------|
| | Numerical | Analytic | |
| 0 | 50 | 50 | 0 |
| 10 | 49 | 49 | 0.199 |
| 20 | 44 | 44 | 0.389 |
| 30 | 41 | 41 | 0.565 |
| 40 | 34 | 34 | 0.717 |
| 50 | 27 | 27 | 0.841 |
| 60 | 18 | 18 | 0.932 |
| 70 | 8 | 8.5 | 0.985 |
| 75 | 2 | 3.5 | 0.997 |

Time evolution of the loop radius in unit of Δx . The third column shows analytic estimate based on Eq. (8) and the fourth the speed of the loop motion (9).

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